

## Exercise 18

For the following exercises, use synthetic division to find the quotient.

$$(9x^3 - 9x^2 + 18x + 5) \div (3x - 1)$$

### Solution

Synthetic division only works if the divisor is of the form  $x - k$ . Multiply the numerator and denominator by  $1/3$  to make it so.

$$\frac{9x^3 - 9x^2 + 18x + 5}{3x - 1} = \frac{9x^3 - 9x^2 + 18x + 5}{3x - 1} \times \frac{1/3}{1/3} = \frac{3x^3 - 3x^2 + 6x + 5/3}{x - 1/3}$$

Solving  $x - \frac{1}{3} = 0$  gives  $x = \frac{1}{3}$ ; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & 3 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & 3 & 1 & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{c|ccc|c} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & & & 1 & \\ \hline & 3 & -2 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{c|ccc|c} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & & & 1 & -\frac{2}{3} \\ \hline & 3 & -2 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{c|ccc|c} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ \hline & & & 1 & -\frac{2}{3} \\ \hline & 3 & -2 & \frac{16}{3} & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} \\ & & & 1 & -\frac{2}{3} & \frac{16}{9} \\ \hline & 3 & -2 & \frac{16}{3} & & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{1}{3} & 3 & -3 & 6 & \frac{5}{3} & \\ & & & 1 & -\frac{2}{3} & \frac{16}{9} \\ \hline & 3 & -2 & \frac{16}{3} & \frac{31}{9} & \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is  $3x^2 - 2x + \frac{16}{3}$ .

$$(9x^3 - 9x^2 + 18x + 5) \div (3x - 1) = 3x^2 - 2x + \frac{16}{3} + \frac{\frac{31}{9}}{x - \frac{1}{3}}$$